Continuous Random Variables

Lecture 23
Section 7.5.4

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Outline

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Introduction

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- How is its value distributed over the many possible samples?

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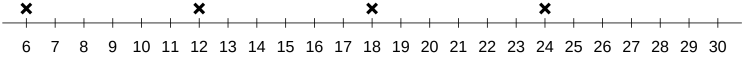
- If we make several observations, then we need to use a statistic, such as the average, for our decision.
- In that case, we need to know the *distribution* of that statistic when H_0 is true.
- How is its value distributed over the many possible samples?
- How does the distribution change when the sample size increases?

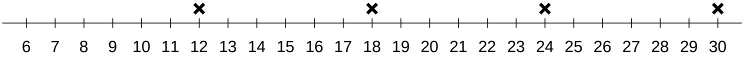
Outline

Bag A vs. Bag B

- Suppose Bag A contains vouchers worth 6, 12, 18, and 24 dollars.
- And Bag B contains vouchers worth 12, 18, 24, and 30 dollars.
- We are handed one bag.
- The two hypotheses:
  H_0 : It is Bag A.
  H_1 : It is Bag B.
- We select one voucher at random.
- If its value is greater then $18, we will reject H_0.

Sample Size 1





What are α and β?

Sample Size 2

- Now we select two vouchers and get their average.
- If the average is greater then \$18, we will reject H_0.

Sample Size 2

	6	12	18	24
6	6	9	12	15
12	9	12	15	18
18	12	15	18	21
24	15	18	21	24

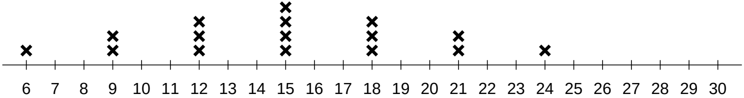
Sample means in Bag A, with $n = 2$

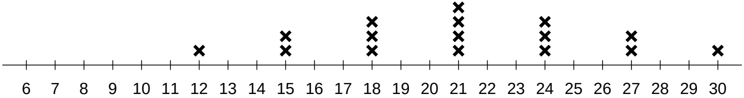
Sample Size 2

	12	18	24	30
12	12	15	18	21
18	15	18	21	24
24	18	21	24	27
30	21	24	27	30

Sample means in Bag B, with $n = 2$

Sample Size 2





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Sample Size 3

- Now we select three vouchers and get their average.
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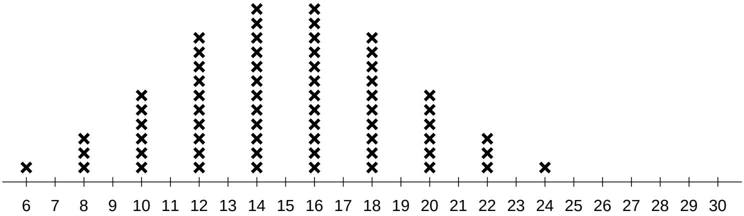
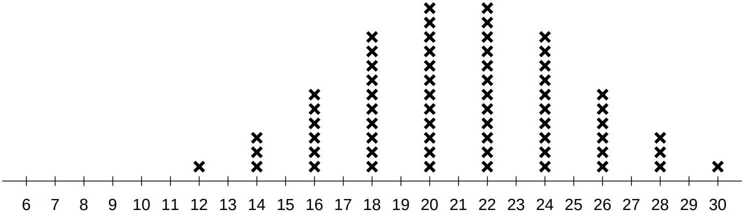
- If the average is greater then \$18, we will reject H_0.
- We will dispense with the extensive calculations and go straight to the graphs.

Sample Size 3





What are α and β?

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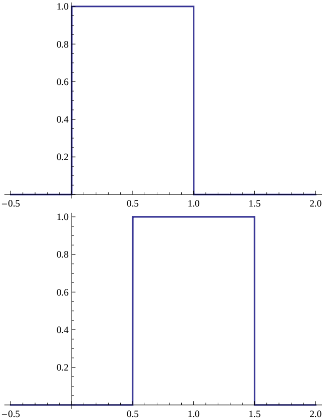
Hypothesis Testing ($n = 1$)

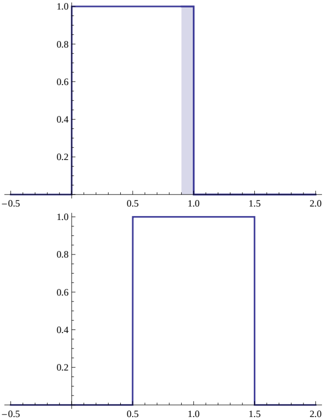
- An experiment is designed to determine whether a random variable X has the distribution $U(0, 1)$ or $U(0.5, 1.5)$.
  - $H_0 : X$ is $U(0, 1)$.
  - $H_1 : X$ is $U(0.5, 1.5)$.
- One value of X is sampled ($n = 1$).
- If X is more than 0.90, then H_0 will be rejected.

Hypothesis Testing ($n = 1$)

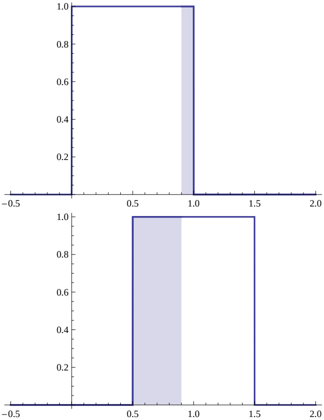


Hypothetical distributions of X under H_0 and H_1:

Hypothesis Testing ($n = 1$)



What is α?



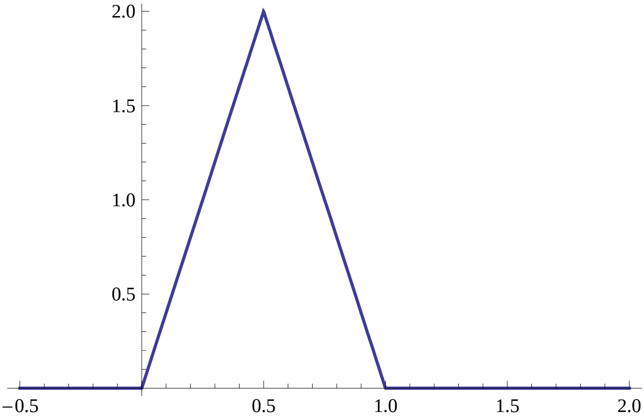
What is β?

Outline

Example

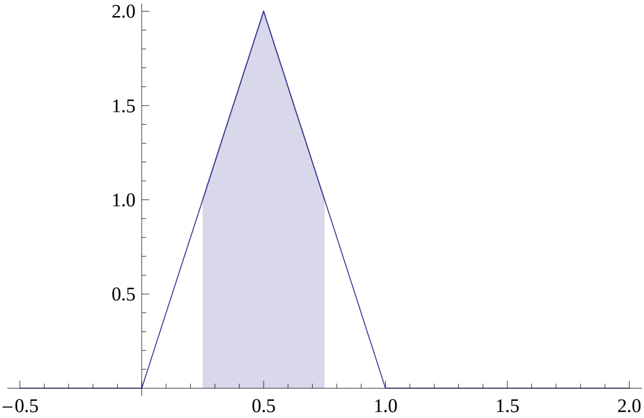
- Now suppose we use the TI-83 to get *two* random numbers from 0 to 1.
- Let $X_2 =$ the average of the two random numbers.
- What is the pdf of X_2?

Example



The graph of the pdf of X_2.

Example

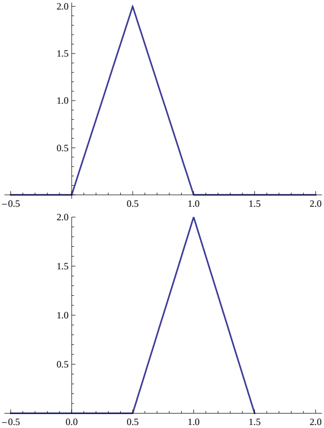


What is the probability that X_2 is between 0.25 and 0.75?

Hypothesis Testing ($n = 2$)

- An experiment is designed to determine whether a random variable X has the distribution $U(0, 1)$ or $U(0.5, 1.5)$.
  - $H_0 : X$ is $U(0, 1)$.
  - $H_1 : X$ is $U(0.5, 1.5)$.
- Two values of X are sampled ($n = 2$).
- Let X_2 be the average.
- If X_2 is more than 0.90, then H_0 will be rejected.

Hypothesis Testing ($n = 2$)



Hypothetical distributions of X under H_0 and H_1:

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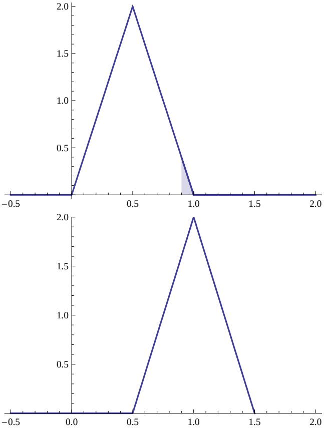
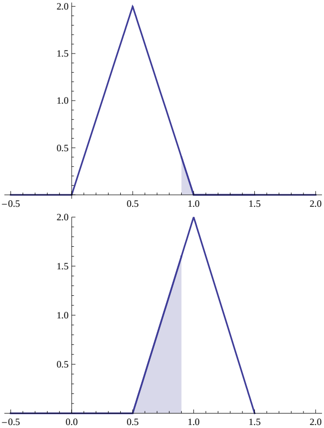


What is α?

Hypothesis Testing ($n = 2$)



What is β?

Conclusion

Conclusion

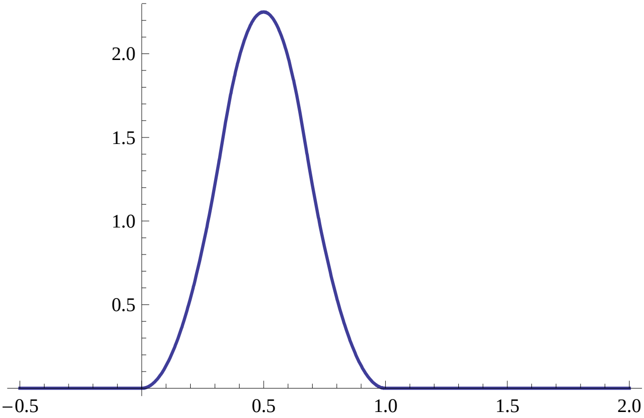
- By increasing the sample size, we can lower both α and β simultaneously.

Outline

Example

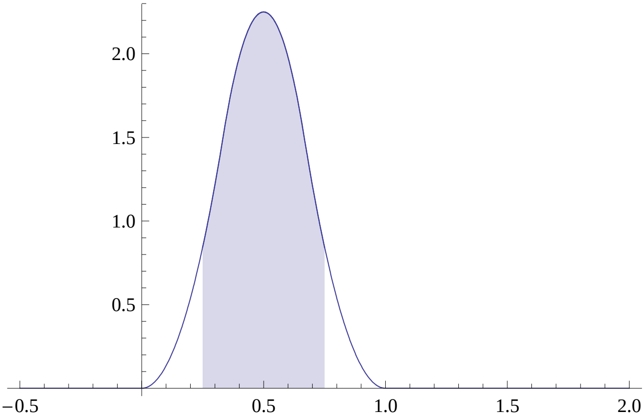
- Now suppose we use the TI-83 to get *three* random numbers from 0 to 1, and then average them.
- Let $X_3 =$ the average of the three random numbers.
- What is the pdf of X_3?

Example



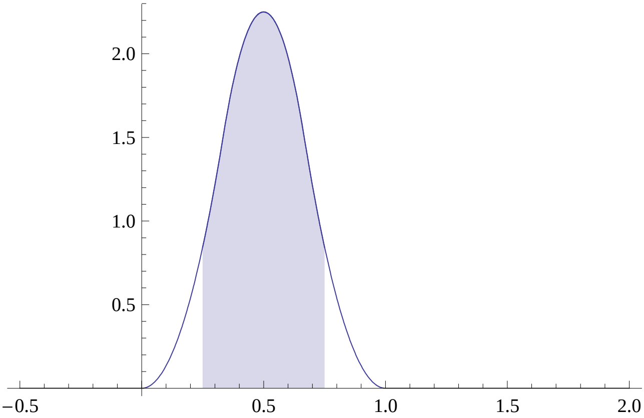
The graph of the pdf of X_3.

Example



What is the probability that X_3 is between 0.25 and 0.75?

Example

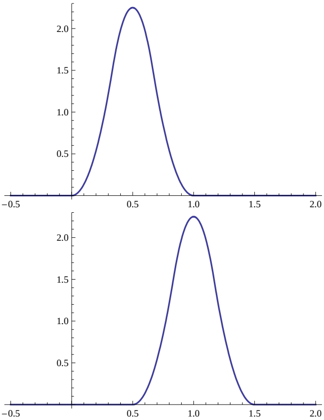


It turns out to be $\frac{55}{64} = 0.8954$.

Hypothesis Testing (n = 3)

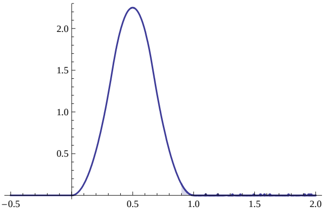
- An experiment is designed to determine whether a random variable X has the distribution $U(0, 1)$ or $U(0.5, 1.5)$.
  - H_0 : X is $U(0, 1)$.
  - H_1 : X is $U(0.5, 1.5)$.
- Three values of X_3 are sampled ($n = 3$). Let X_3 be the average.
- If X_3 is more than 0.90, then H_0 will be rejected.

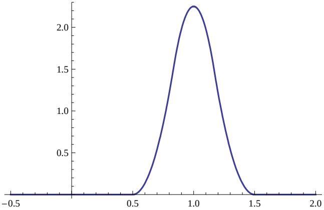
Hypothesis Testing (n = 3)



The hypothetical distributions of X_3 under H_0 and H_1.

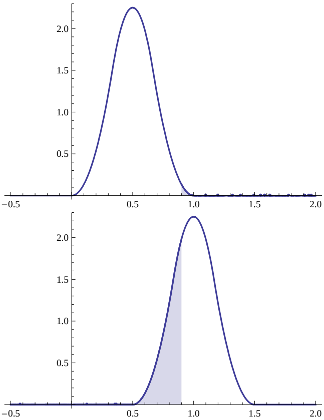
Hypothesis Testing (n = 3)





It turns out that $\alpha = 0.0045$.

Hypothesis Testing (n = 3)



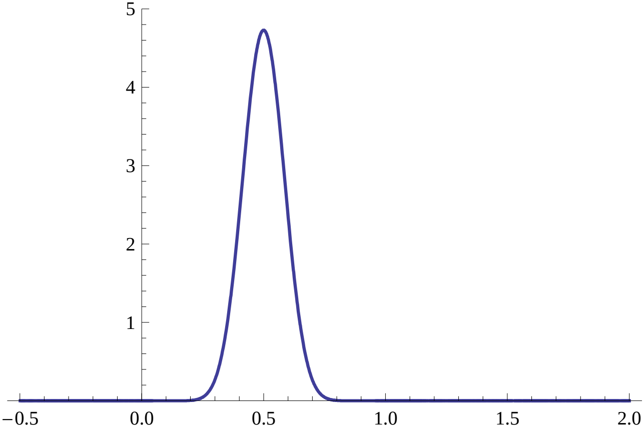
And $\beta = 0.2840$.

Outline

Example

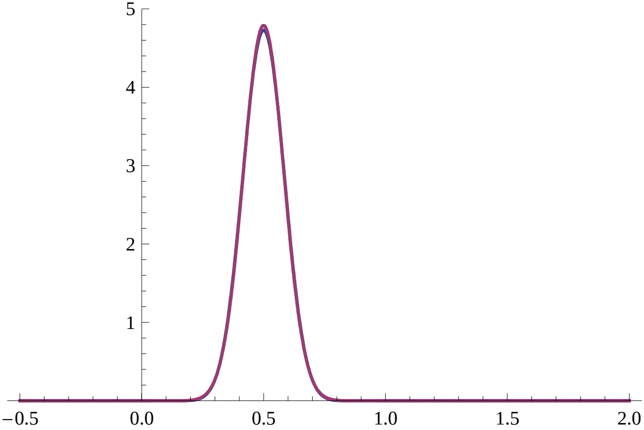
- Suppose we get 12 random numbers, uniformly distributed between 0 and 1, from the TI-83 and get their average.
- Let $X_{12} =$ average of 12 random numbers from 0 to 1.
- What is the pdf of X_{12}?

Example



It turns out that the pdf of X_{12} is nearly exactly normal with a mean of $\frac{1}{2}$ and a standard deviation of $\frac{1}{12}$.

Example



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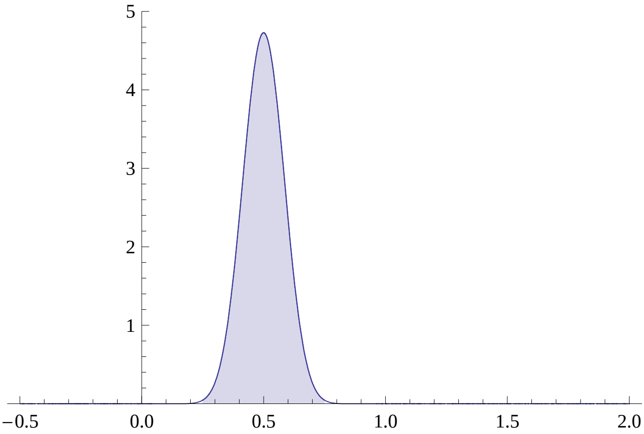
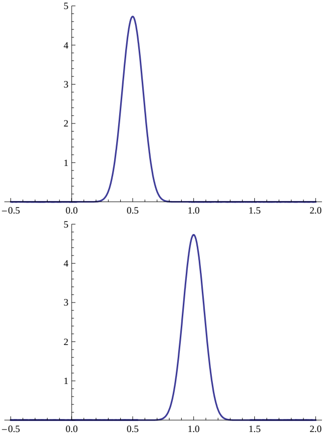


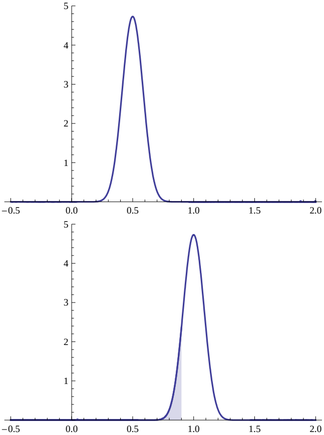
What is the probability that X_{12} is between 0.25 and 0.75?

Hypothesis Testing (n = 12)



The hypothetical distributions of X_{12} under H_0 and H_1.

Hypothesis Testing (n = 12)



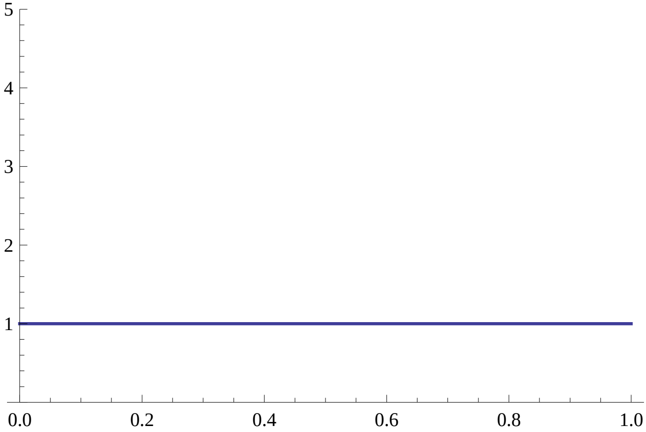
What are α and β?

Outline

Preview of the Central Limit Theorem

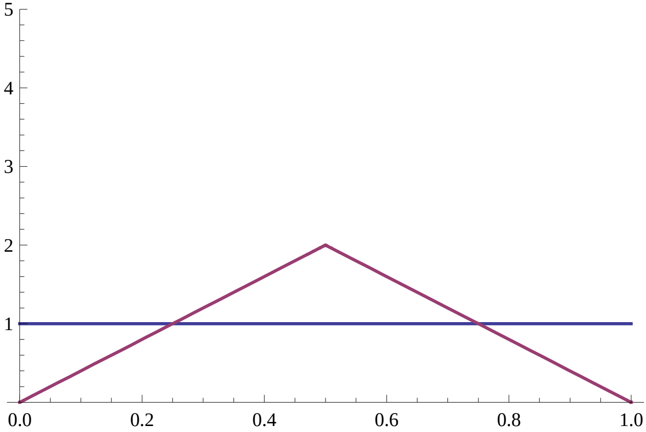
- We looked at the distribution of the average of 1, 2, 3, and 12 uniform random variables $U(0, 1)$.
- We saw that the shapes of their distributions was moving towards the shape of the normal distribution.

Preview of the Central Limit Theorem



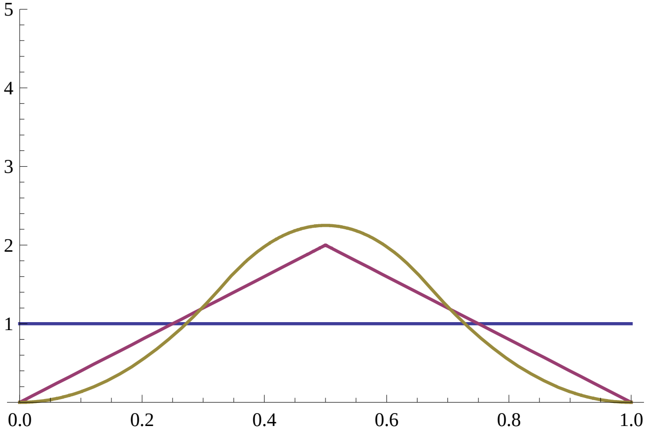
$n = 1$

Preview of the Central Limit Theorem

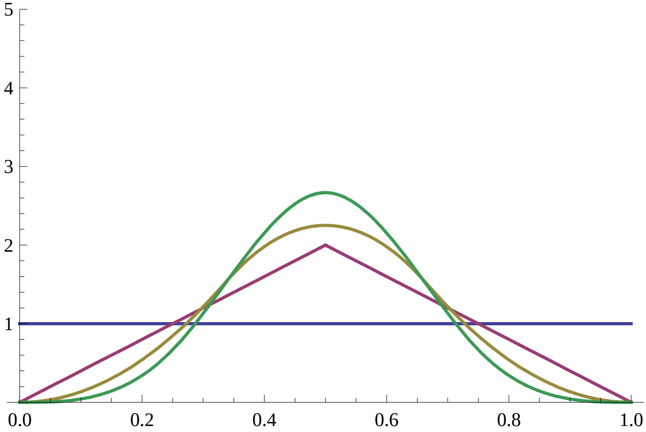


$n = 2$

Preview of the Central Limit Theorem

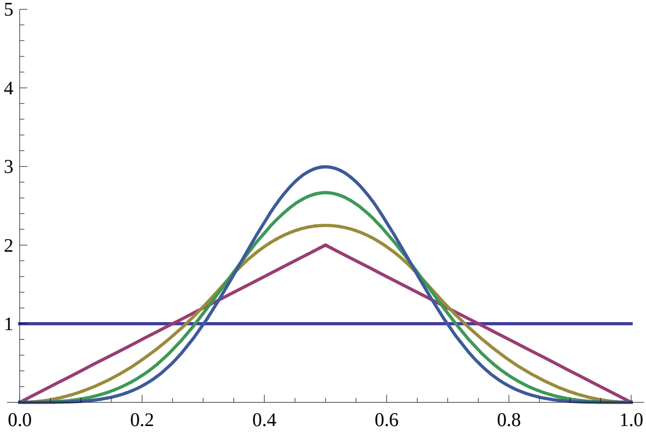


$n = 3$



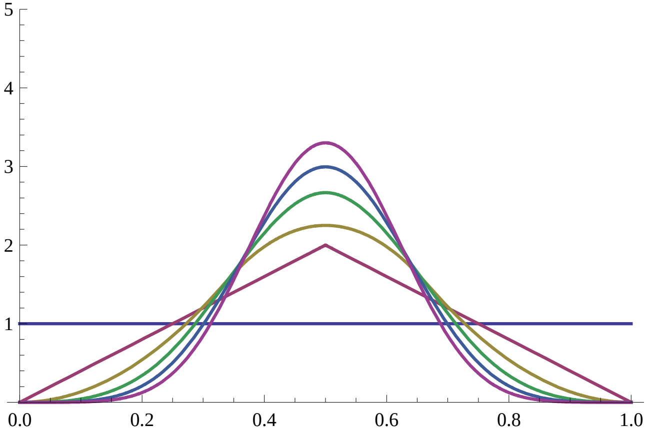
$n = 4$

Preview of the Central Limit Theorem



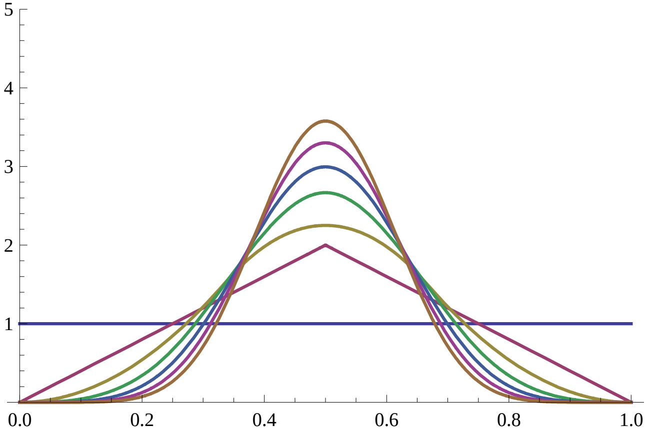
$n = 5$

Preview of the Central Limit Theorem



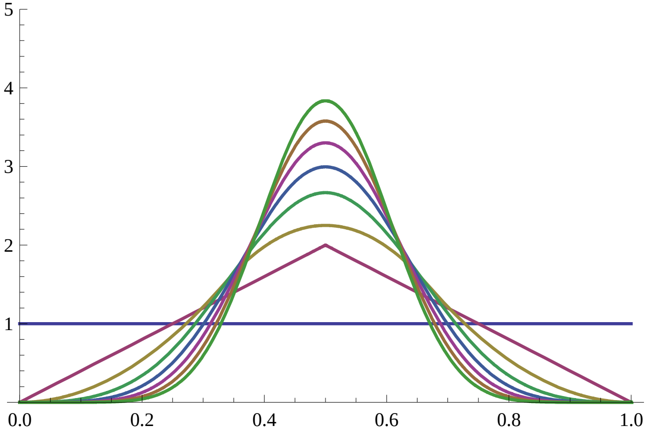
$n = 6$

Preview of the Central Limit Theorem



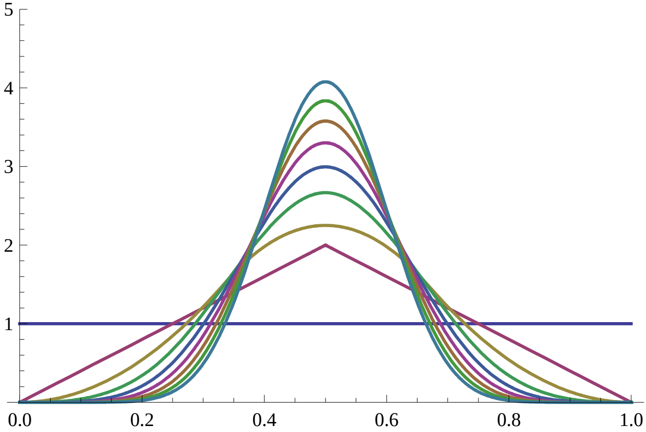
$n = 7$

Preview of the Central Limit Theorem



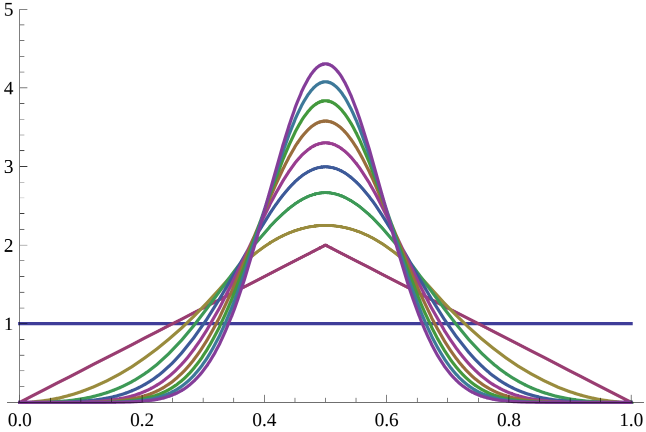
$n = 8$

Preview of the Central Limit Theorem



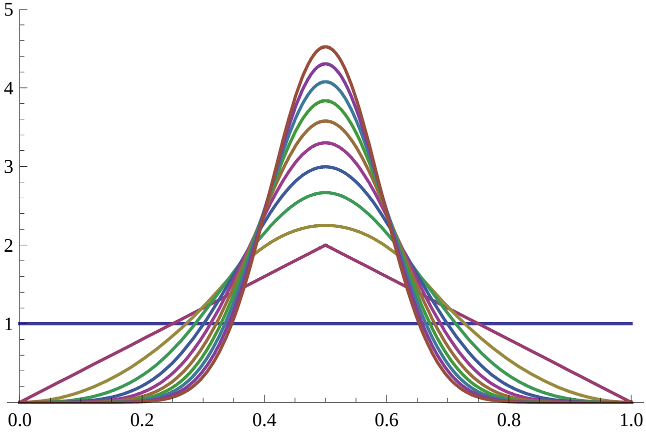
$n = 9$

Preview of the Central Limit Theorem



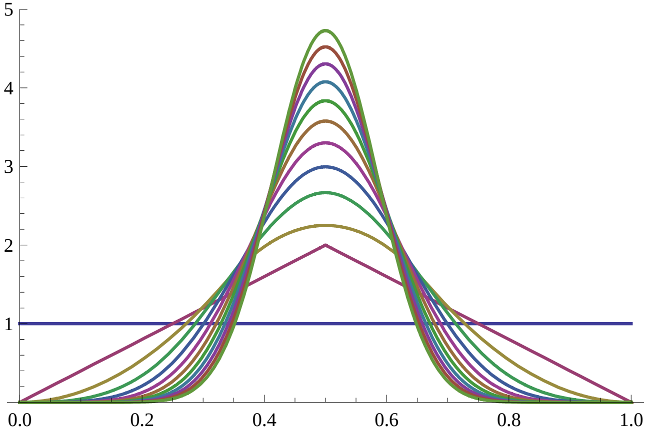
$n = 10$

Preview of the Central Limit Theorem



$n = 11$

Preview of the Central Limit Theorem



$n = 12$

Preview of the Central Limit Theorem

- Some observations:
  - Each distribution is centered at the same place, $\frac{1}{2}$.
  - The distributions are being “drawn in” towards the center.
  - That means that their standard deviation is decreasing as the sample size increases.
- Can we quantify this?

Outline

Assignment

Homework

- Review Exercises 72, 104, 105ab, 106, page 489.